Synthesis of an Adaptive Controller for Movement Synchronization in a Multi-channel Steering System

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***Abstract*— The paper presents the synthesis of an adaptive controller for a redundant steering drive system, consisting of several electro-hydraulic actuators that control a common steering surface. A mathematical model of an electro-hydraulic actuator system is analyzed taking into account nonlinear factors. The construction of the synchronization control purpose function is presented, from which the adaptive control law for equalizing the loading forces of the channels is built on the basis of the method of Lyapunov function. The research results were performed in the MATLAB / Simulink software environment when comparing the response of the output force between channels with different synchronization coefficients of the control law.**

Keywords—synchronization algorithms; redundant steering system; hybrid steering drives; adaptive control

I. INTRODUCTION

One of the key trends in modern civil aircraft construction is increasing requirements for the reliability of flight control actuators. The problem is becoming increasingly important since aircraft have become larger and more complex. Therefore, structural redundancies have been introduced into the flight control system to increase the reliability of the executive system (Fig. 1).

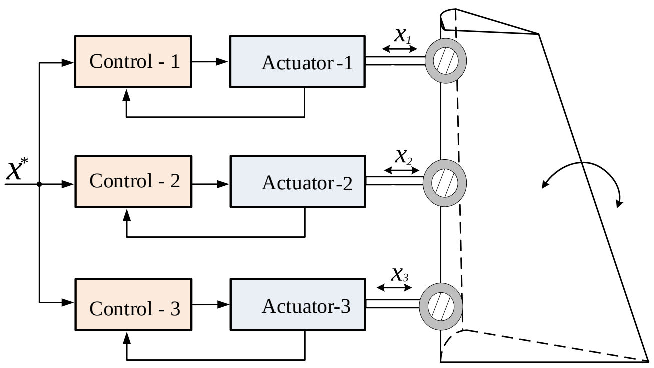

Fig. 1. Multi-channel redundant steering system

Obviously, if the actuators are directly connected to the steering surface through elastic connections with high connection rigidity, this leads to a coupling effect between the outputs of the actuators. These features lead to a serious problem of mutual loading of the actuators and, as a consequence, a decrease in the control accuracy of the aircraft's control surface or even damage to the steering surface. Therefore, the task of reducing mutual loading forces using adaptive control and synchronization algorithms under conditions of limited uncertainty of system parameters is an urgent task for controlling multi-channel steering actuators of civil aircraft.

An approach to solving this problem is to construct a function of synchronization goals, from which we have proven that the state variables of the channels will converge to the value of the state vector, which is the average of all channels in the system. The adaptive synchronization law is chosen in such a way that the synchronous purpose function satisfies the convergence criteria based on the Lyapunov function.

II. CONSTRUCTION OF A MATHEMATICAL MODEL OF MULTI-CHANNEL REDUNDANT STEERING SYSTEMS

We are considering directional steering systems in an aircraft [1], which consists of three electrohydraulic actuator (EHA) with throttle speed control, having the same configuration and operating in the mode of summing the forces they develop on a common output link – the control surface (Fig. 1). The kinematic diagram of the electrohydraulic steering drive of each channel is shown in Fig. 2.

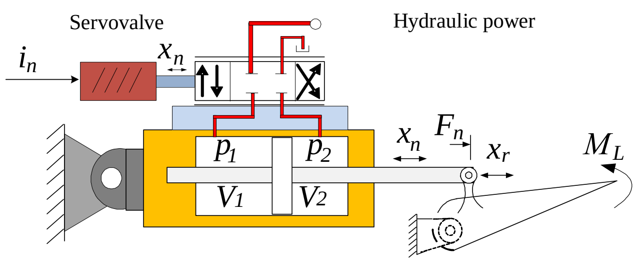

Fig. 2. EHA scheme

The mathematical model of each channel is described by the following equations [1,2,3,4]:

- The equation describes the movement of the servo valve spool linearly related to the servo valve current:

$$\tau_n \dot{x}_n = -x_n + k_n i_n \tag{1}$$

- The equation describes the flow rate of load Q_n related to the displacement of spool valve x_n and load pressure P_n:

$$Q_n = \mu_n \sqrt{\frac{1}{\rho_n}}\, b_n x_n \sqrt{(P_s - P_n \operatorname{sgn}(x_n))} \tag{2}$$

- The equations for the balance of fluid flow taking into account its compressibility and leakage and the equation of motion of the hydraulic cylinder have the form (3) and (4), respectively:

$$Q_n = A_n\dot{x}_n + \frac{V_n}{2E_n}\dot{P}_n + C_n P_n \quad (3)$$

$$A_n P_n = m_n\ddot{x}_n + B_n\dot{x}_n + F_n \quad (4)$$

- The mathematical model of the control surface [2,3,4] is described by equations (5) and (6):

$$F_n = K_n(x_n - x_r) \quad (5)$$

$$F_\Sigma r_r = J_r\ddot{\theta}_r + b_r\dot{\theta}_r + M_L\,. \quad (6)$$

In Fig. 2 and in equations (1) - (6) the following notations are used: n – index of the channel n (n =1,2,3); τ and k – time constant and gain of the electro-hydraulic power amplifier, respectively; i – torque motor current ; P_S – supply pressure; μ – flow coefficient; b – area gradient, relates orifice area to spool displacement; ρ – fluid density; A – piston square area; m – piston rod weight of the hydraulic cylinder; B – viscous friction of the hydraulic cylinder; C – leakage coefficient; x – displacement of EHA; F – force output of EHA; θ_r – a rudder angular position; r_r – a lever to control surface; J_r – an equivalent moment of inertia; K – stiffness coefficient of EHA coupling with a flight control surface; x_r – rudder displacement; b_r – rudder damping coefficient; F_Σ – summation of the forces of three channels on the rudder surface; M_L – external load applied to the rudder.

After linearization of the system (linealization of equation (2)) at operating points, the mathematical description in the state space of each drive channel has the following general form [2]:

$$\dot{\mathbf{x}}_n = \mathbf{A}_n\mathbf{x} + \mathbf{B}_n\mathbf{u}_n + \boldsymbol{\sigma}_n\ ,\ y_n = \mathbf{C}_n\mathbf{x}_n\ , \quad (7)$$

where $\mathbf{B}_n = [0\quad 0\quad 0\quad 0\quad 0\quad k_n/\tau_n]^T$,

$$\mathbf{A}_n = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-r_r^2 K_n}{J_r} & \frac{-b_r}{J_r} & \frac{r_r K_n}{J_r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{K_n r_r}{m_n} & 0 & \frac{-K_n}{m_n} & \frac{-B_n}{m_n} & \frac{A_n}{m_n} & 0 \\ 0 & 0 & 0 & \frac{-2A_n E_n}{V_n} & \frac{-2E_n(C_n + K_{QP})}{V_n} & \frac{2E_i K_{Qx}}{V_n} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_n} \end{bmatrix},$$

$\boldsymbol{\sigma}_n$ – vector function of reduced disturbances; coefficients K_{Qx} and K_{QP} are results of linearization of (2) at operating points ($Q_h = K_{Qx}x_v - K_{QP}P_h$) [3].

III. Synthesis of an Adaptive Controller for a Multi-Channel Steering Drive System

The synthesis of an adaptive controller for each channel is considered based on the reference model in the form of a vector of state variables (for all channels):

$$\dot{\mathbf{x}}_M = \mathbf{A}_M\mathbf{x}_M + \mathbf{B}_M g\ , \quad (8)$$

where $\mathbf{A}_M = \mathbf{A}_n - \mathbf{B}_n\mathbf{K}$;$\mathbf{B}_M = \mathbf{B}_n$; $\mathbf{K}$ – a modal controller coefficient matrix. A state observers are used to assess the state vector of the control plant $\dot{\hat{\mathbf{x}}} = \mathbf{A}_n\hat{\mathbf{x}} + \mathbf{B}_n u + \mathbf{G}(y - \hat{y})$; $\hat{y} = \mathbf{C}\hat{\mathbf{x}}$, where the observer dynamics matrix G is determined by the equation: $\det[p\mathbf{I} - (\mathbf{A} - \mathbf{GC})] = \mathbf{H}_g(p)$.

The cost function, including the objective of synchronizing channel state variables and ensuring the stability of the adaptive system, is defined as follows [6]:

$$\lim_{t\to\infty}\mathbf{E}_n(t) = \lim_{t\to\infty}[\mathbf{k}\Delta\mathbf{e}_n(t) + \mathbf{e}_n(t)] = \boldsymbol{\xi}_n\ , \qquad \|\boldsymbol{\xi}_n\| \le \xi_0\ , \quad (9)$$

where $\Delta\mathbf{e}_n = \mathbf{x}_n - \frac{1}{3}\sum_{i=1}^{3}\mathbf{x}_i$ is the synchronization error (deviation of the state vector of the nth channel from the average value of the state vectors of all channels), $\mathbf{e}_n(t) = \mathbf{x}_n - \mathbf{x}_M$ is the control error, $\mathbf{k} = diag(k_1, k_2, k_3)$ is the synchronization coefficient, ξ_0 is the small number.

Rationale for the cost function (9): Taking into account (9), performing transformation $\lim_{t\to\infty}\mathbf{E}_k(t) - \lim_{t\to\infty}\mathbf{E}_l(t)$ or any two channels with indices k and l ($\forall k, l \in \overline{1,..,3}$) we obtain:

$$\lim_{t\to\infty}[\mathbf{x}_k(t) - \mathbf{x}_l(t)] = (\boldsymbol{\xi}_k - \boldsymbol{\xi}_l)\frac{1}{(\mathbf{I} + \mathbf{k})}\,. \quad (10)$$

From (10) and (9) it follows $\|\Delta\mathbf{e}_n(t)\| \le \xi_0$ and $\|\mathbf{e}_n(t)\| \le \xi_0$ when $t \to \infty$ That is, in static mode, the control goal (9) ensures stability and synchronization in the system.

The cost function must ensure increased system synchronization in dynamic mode, then the cost function (9) at the transition time ($t = t_t$) can be expressed as follows:

$$\lim_{t\to t_t}\mathbf{E}_n = \lim_{t\to t_t}[\mathbf{e}_n + \mathbf{k}\Delta\mathbf{e}_n] = \Delta_t^n\ , \quad (11)$$

where Δ_t^n – is the vector characterizing the control error n-th channel at the time of the system transient process. Then with indices k and l, representing any two channels in the system ($\forall l, k \in \overline{1,...,3}$) from (11) we obtain:

$$\lim_{t\to t_t}(\mathbf{x}_k - \mathbf{x}_l) = \frac{\Delta_t^k - \Delta_t^l}{(\mathbf{I} + \mathbf{k})}\ , \quad (12)$$

where $\mathbf{I}$ – is the identity matrix. From (12) we can conclude that with an increase in the vector components synchronization coefficient $\mathbf{k}$, the magnitudes of error vectors ($\mathbf{x}_k - \mathbf{x}_l$) decrease, which proves that the cost function (9) is fulfilled in a dynamic mode.

Synthesis of an adaptive control law for each channel: To achieve the cost function (9), consider Lyapunov functions of a positive definite form:

$$V(\mathbf{E}_n) = \mathbf{E}_n^T\mathbf{P}_n\mathbf{E}_n \quad (13)$$

where matrix $\mathbf{P}_n = \mathbf{P}_n^T > 0$ – determined from the solutions of the equation $\mathbf{A}_M^T\mathbf{P}_n + \mathbf{P}_n\mathbf{A}_M = -\mathbf{Q}_n$, $\mathbf{Q}_n = \mathbf{Q}_n^T > 0$ – diagonal, positive definite matrices.

To achieve the goal of the synchronization control (9), it is necessary to synthesize the control law of the following form [1,2,8]:

$$u_n = g + z_n, \tag{14}$$

for channel n , so that the condition $\dot{V}(\mathbf{E}_n) < 0$ is satisfied. Where g – is a program control signal, z_n – is an adaptive control signal for channel n, including components of synchronization and adaptation signals.

Considering (8) in the form, equation (7) would be rewritten as follows [1,7]:

$$\dot{\mathbf{x}}_n = \mathbf{A}_M \mathbf{x}_n + \mathbf{B}_M u_n + \boldsymbol{\rho}_n \tag{15}$$

where $\boldsymbol{\rho}_n = (\mathbf{A}_n - \mathbf{A}_M)\mathbf{x}_n + (\mathbf{B}_n - \mathbf{B}_M)u_n + \boldsymbol{\sigma}_n$ - is the vector of reduced disturbances of the channel n.

From the equation:

$$\mathbf{E}_n(t) = \mathbf{e}_n(t) + \mathbf{k}\Delta\mathbf{e}_n = \mathbf{x}_n - \mathbf{x}_M + \mathbf{k}\Delta\mathbf{e}_n \tag{16}$$

we have:

$$\dot{\mathbf{E}}_n(t) = \dot{\mathbf{x}}_n - \dot{\mathbf{x}}_M + \mathbf{k}\Delta\dot{\mathbf{e}}_n . \tag{17}$$

Taking into account (17) from (8) and (15) it turns out:

$$\dot{\mathbf{E}}_n = \mathbf{A}_M \mathbf{e}_n + \mathbf{k}\Delta\dot{\mathbf{e}}_n + \mathbf{B}_M z_n + \boldsymbol{\rho}_n . \tag{18}$$

Then, taking into account (13) from (18) and (16), we obtain:

$$\dot{\mathbf{V}} = [\mathbf{A}_M \mathbf{E}_n - \mathbf{A}_M(\mathbf{k}\Delta\mathbf{e}_n) + \mathbf{k}\Delta\dot{\mathbf{e}}_n + \mathbf{B}_M z_n + \boldsymbol{\rho}_n]^T \mathbf{P}_n \mathbf{E}_n + \\ + \mathbf{E}_n^T \mathbf{P}_n [\mathbf{A}_M \mathbf{E}_n - \mathbf{A}_M(\mathbf{k}\Delta\mathbf{e}_n) + \mathbf{k}\Delta\dot{\mathbf{e}}_n + \mathbf{B}_M z_n + \boldsymbol{\rho}_n] \tag{19}$$

set

$$\eta_n = -\mathbf{B}_M^{+}\mathbf{A}_M(\mathbf{k}\Delta\mathbf{e}_n) + \mathbf{B}_M^{+}\mathbf{k}\Delta\dot{\mathbf{e}}_n , \tag{20}$$

where $\mathbf{B}_M^{+} = (\mathbf{B}_M^{\mathrm{T}}\mathbf{B}_M)^{-1}\mathbf{B}_M^{\mathrm{T}}$ - is a pseudoinverse matrix to $\mathbf{B}$. Then from (20) we have:

$$-\mathbf{A}_M(\mathbf{k}\Delta\mathbf{e}_n) + \mathbf{k}\Delta\dot{\mathbf{e}}_n = \mathbf{B}_M \eta_n . \tag{21}$$

From (19) and taking into account (20) we have:

$$\dot{\mathbf{V}} = -\mathbf{E}_n^T \mathbf{Q}_n \mathbf{E}_n + 2\mathbf{E}_n^T \mathbf{P}_n \mathbf{B}_M (z_n + \eta_n) + 2\mathbf{E}_n^T \mathbf{P}_n \boldsymbol{\rho}_n \tag{22}$$

For system (1), the law of adaptive synchronization control for the n-th channel is specified in the form

$$z_n = -h_n \mathrm{sign}(\mathbf{B}_M^T \mathbf{P}_n \mathbf{E}_n) - \eta_n \tag{23}$$

or

$$z_n = -h_n \mathrm{sign}(\mathbf{B}_m^T \mathbf{P}_n \mathbf{E}_n) + \mathbf{B}_M^{+}\mathbf{A}_M(\mathbf{k}\Delta\mathbf{e}_n) - \mathbf{B}_M^{+}\mathbf{k}\Delta\dot{\mathbf{e}}_n , \tag{24}$$

where $h_n \geq \left\|\mathbf{B}_M^{+}\right\| \sup \left\|\boldsymbol{\rho}_n\right\|$. Then from (22) and (23), we obtain:

$$\Rightarrow \dot{\mathbf{V}}(\mathbf{E}_n) \leq -\mathbf{E}_n^T \mathbf{Q} \mathbf{E}_n - \left|2\mathbf{E}_n^T \mathbf{P}_n \mathbf{B}_M\right| (h_n - \left|\mathbf{B}_M^{+}\boldsymbol{\sigma}_n\right|) \leq 0 \tag{25}$$

From (25) and (13) it follows

$$\lim_{t\to\infty} \mathbf{E}_n(t) \to 0 , \tag{21}$$

that the adaptive synchronous control signal for the channel n has the following complete form:

$$u_n = -h_n \mathrm{sign}(\mathbf{B}_m^T \mathbf{P}_n \mathbf{E}_n) + \mathbf{B}_M^{+}\mathbf{A}_M(\mathbf{k}\Delta\mathbf{e}_n) - \mathbf{B}_M^{+}\mathbf{k}\Delta\dot{\mathbf{e}}_n + g . \tag{22}$$

IV. SIMULATION RESULTS

The simulation results were performed in the MATLAB/ Simulink software environment with using the following parameters [9]: $k = 3.04\,10^{-3}$ m/A ; $\tau = 0.01\,s$; $K_{Qp} = 1.75\,10^{-11}$ Pa m^3/s ; $A = 2.3\times10^{-3}$ m^2 ; $V = 1.1\times10^{-4}$ m^3 ; $E = 8\,10^8$ Pa ; $C = 10^{-11}$ Pa m^3/s ; $m = 25\,kg$; $b = 10^4$ N s/m ; $b_r = 51.75$ N s/m ; $r_r = 0.54$ m ; $J_r = 13.5\,kg$ m^2; $K = 4\,10^8$ N/m .

The results of the study were carried out by comparing the response of the output force (Fig. 3 and 4) between the channels with a step input control action. Assuming that the reason for the difference in load force between channels is due to the difference in the parameters of the actuators in the channels, for example: $K_{Qx1} = 2.7\,m^2$/c ; $K_{Qx2} = 2.8\,$m^2/s ; $K_{Qx3} = 2.9\,$m^2/s . Then figure 3 shows the results of the force difference between the three channels of the steering system in the absence of a synchronous control signal.

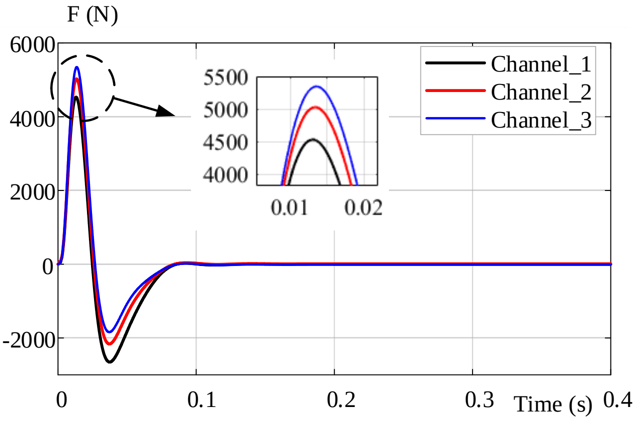

Fig. 3. Reactions of the output forces of channels in the absence of a synchronization algorithm

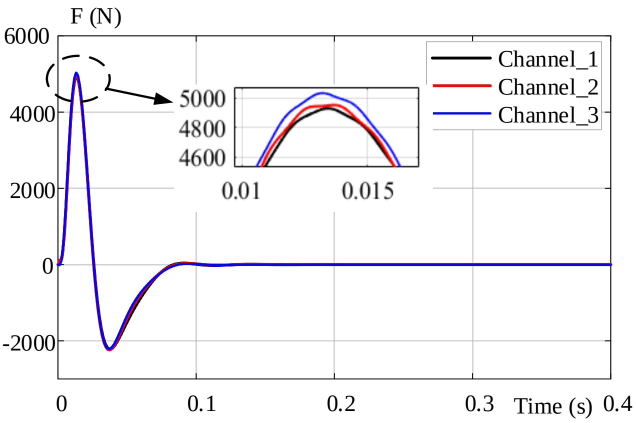

Fig. 4. Channel output force reactions with synchronization algorithm

The effectiveness of the synchronization algorithm is confirmed when the force distribution along the three channels of the system (Fig. 4) is closer to each other than when the synchronization algorithm is not applied (Fig. 3).

V. CONCLUSION

An algorithm has been synthesized for synchronizing the working process between the channels of a hybrid steering drive system, which simultaneously achieves two important goals during the operation of the system, namely:

- provides adaptive stability for each control channel, taking into account nonlinearity in the system;
- provides a significant reduction in output force deviation between channels when the system operates in both dynamic and static modes.

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